TALASIĆI (WAVELETS)

1. Transformacija
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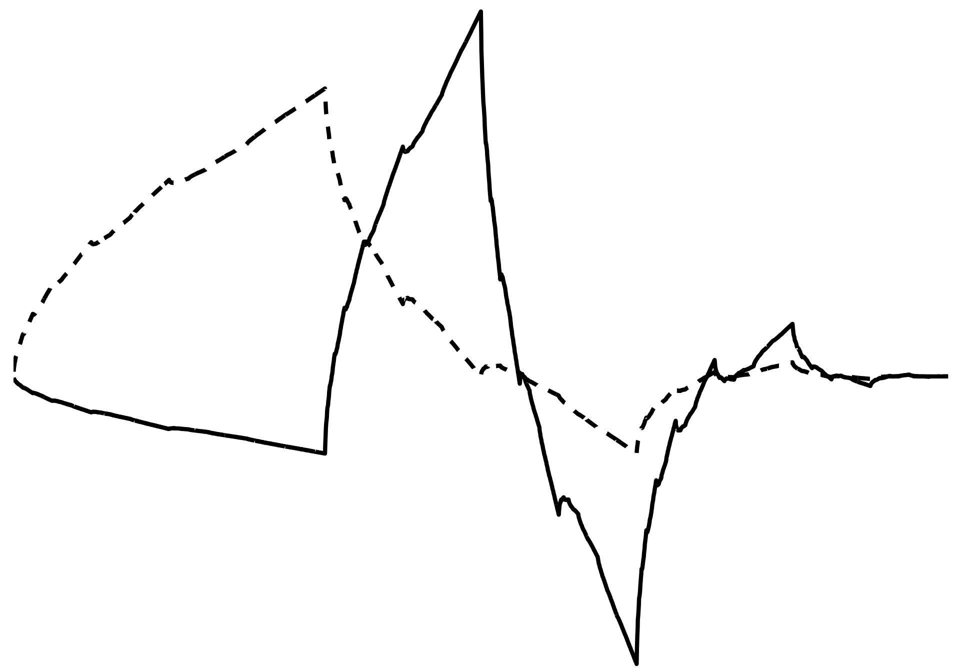

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Multirezolucija prostora $\mathcal{L}_2$

(a) $$\cdots \subset \mathcal{V}_2 \subset \mathcal{V}_1 \subset \mathcal{V}_0 \subset \mathcal{V}_{-1} \subset \mathcal{V}_{-2} \subset \dots$$

(b) $$\cap_{j \in Z} \mathcal{V}_j = \{0\}, \qquad \overline{\cup_{j \in Z} \mathcal{V}_j} = \mathcal{L}_2(R)$$

(c) $\forall f \in \mathcal{L}_2(R)$ i $\forall j \in Z,$ $\quad f(x) \in \mathcal{V}_j \quad \Leftrightarrow \quad f(2x) \in \mathcal{V}_{j-1}$

(d) $\forall f \in \mathcal{L}_2(R)$ i $\forall k \in Z,$ $\quad f(x) \in \mathcal{V}_0 \quad \Leftrightarrow \quad f(x-k) \in \mathcal{V}_0$

(e) $\exists \varphi \in \mathcal{V}_0$ tako da je $\{\varphi(x-k)\}_{k \in Z}$ Riesz-ov bazis u $\mathcal{V}_0$.

$\varphi_{j,k}(x) = 2^{-j/2}\varphi(2^{-j}x - k), \quad j,k \in Z;$ $\qquad \{\varphi_{j,k}(x)\}_{k \in Z}$ Riesz-ov bazis u $\mathcal{V}_j$

Dilataciona jednačina $$\varphi(x) = \sum_{k=0}^{N-1} c(k)\sqrt{2}\varphi(2x-k), \qquad \int \varphi(x)\,dt = 1$$

Prostor talasića $\mathcal{W}_j$: $\mathcal{V}_{j-1} = \mathcal{V}_j \oplus \mathcal{W}_j, \quad j \in Z$

Talasić "*majka*" $\psi(x) \in \mathcal{W}_0$ definisan je *jednačinom talasića*

$$\psi(x) = \sum_{k=0}^{N-1} d(k)\sqrt{2}\varphi(2x-k)$$

$$\psi_{j,k}(x) = 2^{-j/2}\psi(2^{-j}x-k) \quad k \in Z; \qquad \{\psi_{j,k}(x)\}_{k\in Z} \text{ bazis u } \mathcal{W}_j$$

Multirezolucijski razvoj

$$\mathcal{V}_{j-1} = \quad \mathcal{V}_j \quad \oplus \quad \mathcal{W}_j \qquad f_j(x) = \sum_{k\in Z}(f,\, \varphi_{j,k})\,\varphi_{j,k}(x),$$

$$f_{j-1}(x) = f_j(x) + \Delta f_j(x) \qquad \Delta f_j(x) = \sum_{k\in Z}(f,\, \psi_{j,k})\,\psi_{j,k}(x)$$

$$\mathcal{V}_{j-1} = \mathcal{V}_J \oplus \mathcal{W}_J \oplus \cdots \oplus \mathcal{W}_{j+1} \oplus \mathcal{W}_j, \qquad J > j$$

$$\begin{aligned} f_{j-1}(x) &= f_J + (f_{J-1} - f_J) + \cdots + (f_{j-1} - f_j) \\ &= f_J(x) + \Delta f_J(x) + \cdots + \Delta f_j(x) \end{aligned}$$

Granični slučaj $j \to -\infty$ $\qquad \mathcal{L}_2(R) = \mathcal{V}_J \oplus \sum_{j=-\infty}^{J} \mathcal{W}_J$

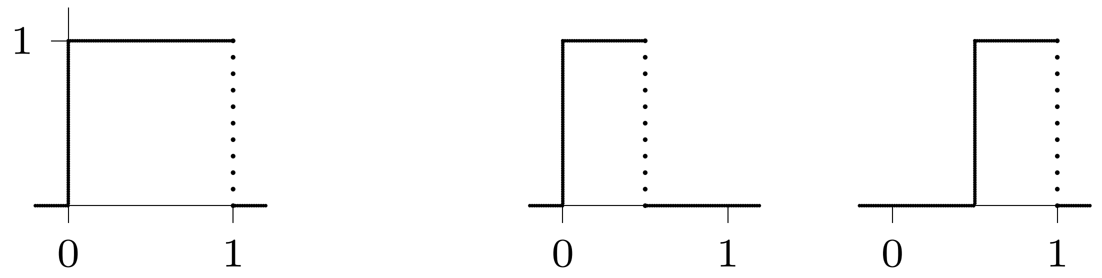
$$f(x) = f_J(x) + \sum_{j=-\infty}^{J} \Delta f_j(x), \qquad f(x) = \sum_{j=-\infty}^{\infty} \Delta f_j(x) \quad (J \to \infty)$$

$$f(x) = \sum_{k \in \mathcal{Z}} a_{J,k}\, \varphi_{j,k}(x) + \sum_{j=-\infty}^{J} \sum_{k \in \mathcal{Z}} b_{j,k}\, \psi_{j,k}(x) = \sum_{j \in \mathcal{Z}} \sum_{k \in \mathcal{Z}} b_{j,k}\, \psi_{j,k}(x)$$

$$\mathcal{V}_J \quad \oplus \quad \mathcal{W}_J \oplus \mathcal{W}_{J-1} \oplus \cdots$$

$$\varphi(x) = \sum_{k \in \mathcal{Z}} c(k)\sqrt{2}\varphi(2x-k), \qquad \psi(x) = \sum_{k \in \mathcal{Z}} d(k)\sqrt{2}\varphi(2x-k)$$

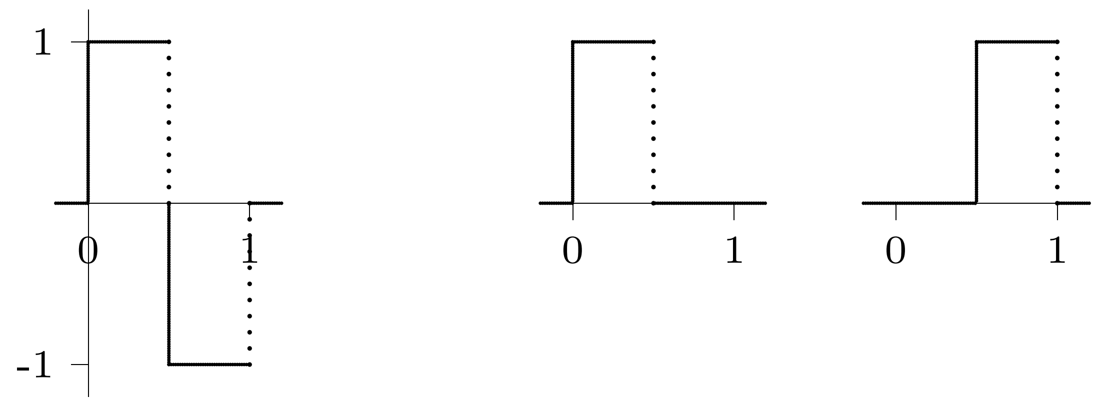
$$\varphi(x) \quad = \quad \varphi(2x) \quad + \quad \varphi(2x-1)$$



Haar-ova četvrtka

$$c(0) = c(1) = \frac{1}{\sqrt{2}}$$

$$\psi(x) \quad = \quad \varphi(2x) \quad - \quad \varphi(2x-1)$$

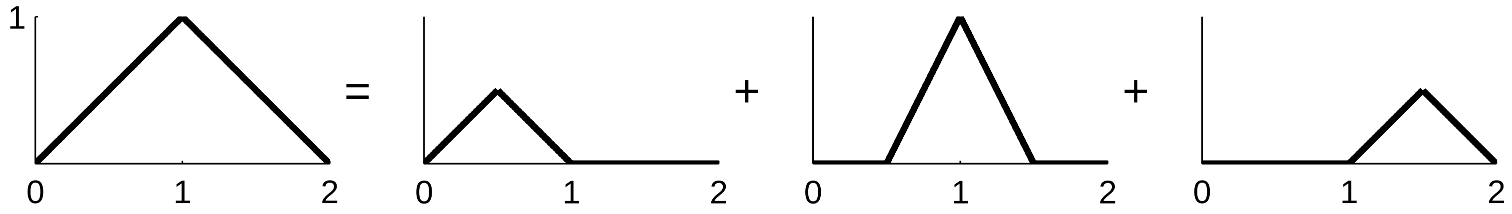


Haar-ov talasić

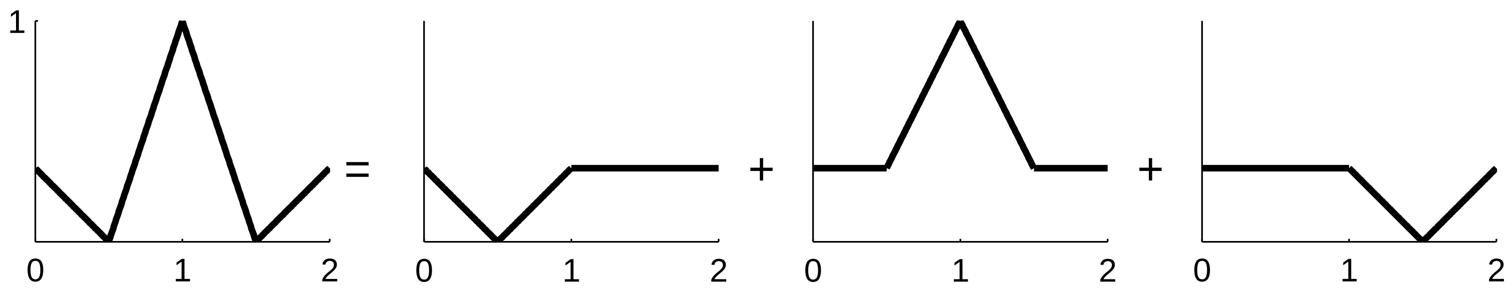
$$d(0) = -d(1) = \frac{1}{\sqrt{2}}$$

Krov funkcija (linearni splajn)

$$\varphi(x) \quad = \quad \frac{1}{2}\varphi(2x) \quad + \quad \varphi(2x-1) \quad + \quad \frac{1}{2}\varphi(2x-2)$$



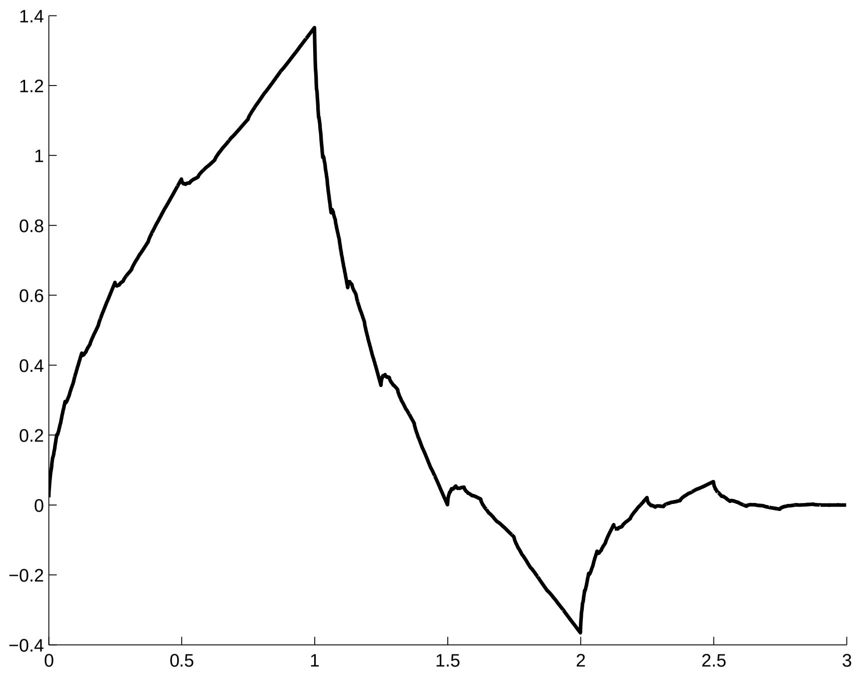
$$\psi(x) \quad = \quad -\frac{1}{2}\varphi(2x) \quad + \quad \varphi(2x-1) \quad - \quad \frac{1}{2}\varphi(2x-2)$$

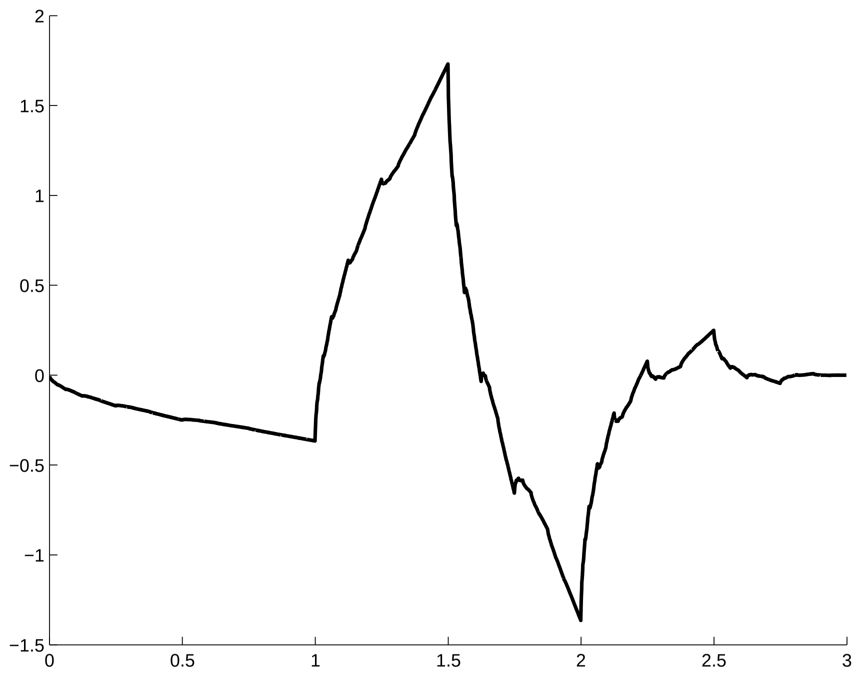


Daubechies Db2 funkcija skaliranja i talasić (ortogonalni sistem)

$$d(0) = c(3) = \frac{1-\sqrt{3}}{4\sqrt{2}}, \qquad d(1) = -c(2) = -\frac{3-\sqrt{3}}{4\sqrt{2}},$$

$$d(2) = c(1) = \frac{3+\sqrt{3}}{4\sqrt{2}}, \qquad d(3) = -c(0) = -\frac{1+\sqrt{3}}{4\sqrt{2}}$$





http://www.matf.bg.ac.yu/ dradun/talasic1024.html